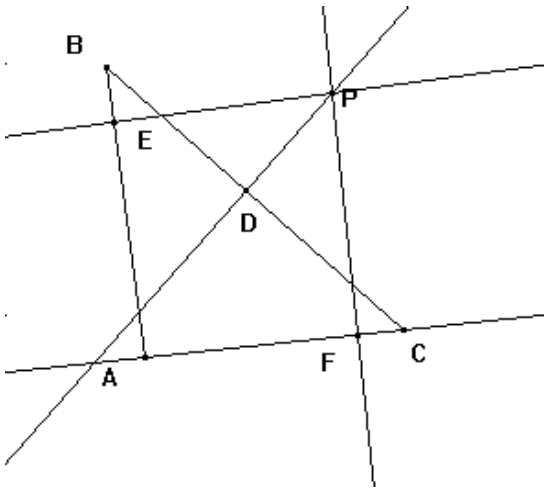
Pedal Triangles

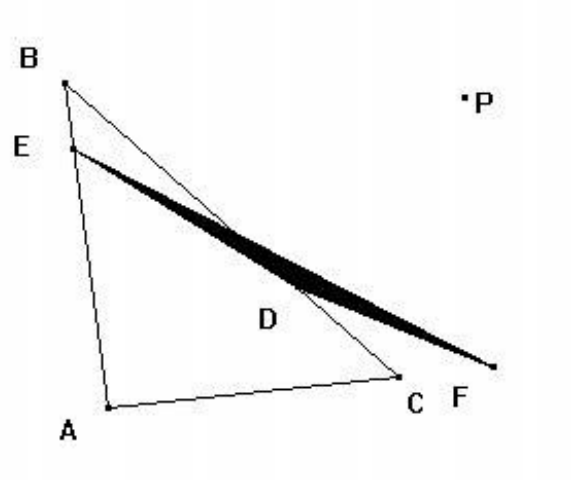
Explore Pedal triangle. Use GSP for the activity.

1. Construct triangle ΔABC and connect each of the other sides with segments.
2. Choose a point P outsite triangle ΔABC.
3. Construct perpendicular lines through point P to each of the sides (AB, BC, and CA). (Note: Extend the sides if necessary).
4. Where the perpendicular lines meets the sides, create point. Those three points are called the feet of a pedal triangle.
5. Label the three points D, E, and F.



6. Construct the triangle ΔDEF, which is the pedal triangle.
7. Define in your own words what a pedal triangle is?

8. Hide the perpendicular lines and other lines.



9. Find the area of the ΔDEF.

10. Drag the point P to make change in area.

11. How large can you make the pedal triangle?

12. How small can you make the pedal triangle?

13. Can the area of the pedal triangle ever be zero? Explain why or why not?

14. Is there a position in which the pedal triangle appears to be equilateral? If so, can you explain why? (Hint: Think about circles).

15. Is there more than one position that it can occur? Why or why not?

16. Can you make the pedal triangle similar to ΔABC?

17. Construct $\overline{AP} = x, \overline{BP} = y, \overline{CP} = z$, and let $\overline{AC} = b, \overline{BC} = a$, and $\overline{AB} = c$

18. Measure $x, y, z, a, b, and\ c$.

19. Find circumcenter of ΔABC.

20. Measure the radius from the circumcenter to $A, B, and\ C$.

21. Can you generalize the formulas for the sides of the pedal triangles (i.e. EF, ED, and DF)? (Hint: Multiply two sides divided by the radius).

22. Summaeize what properties you have discovered about pedal triangles.